You noticed in the introductory audio and video recordings that filters don't completely block frequencies we want removed. Instead, the amplitude of the unwanted frequencies starts to drop off at some point and keeps dropping as we move further away. Any two people listening to the output of a particular audio filter would have different opinions as to when the volume started dropping and when they consider the output to be "quiet enough" to be filtered out, and different filters will drop off at different rates as the frequency moves away from the starting point.

As a result of these issues, the scholarly community has come up with standardized ways of defining what is referred to as the cutoff frequency, as well as the roll-off rate.

Mathematically, the cutoff frequency, $f_c$, is defined as the point at which the power of the signal drops to half what it was in the pass band. This results in a few different expressions that all mean the same thing:

$$P_{fc} = \frac{P_{pass}}{2}$$

Since $P=V^2/R$,

$$V_{fc}{}^2 = \frac{V_{pass}{}^2}{2}$$

which reduces to the most important empirical version of this formula (i.e. based on measured values):

$$V_{fc} = \frac{V_{pass}}{\sqrt{2}}$$

In decibels, 10log(.5) = 3 dB, so the cutoff frequency happens when the signal drops by 3 dB.

Also, this is the point at which $|X_c|$ = R for a capacitive filter. This results in

$$R = \frac{1}{2\ \pi f_c C}$$

which can be rearranged to the most important theoretical version of this formula (i.e. based on component values):

$$f_c = \frac{1}{2\ \pi RC}$$

The rate at which the voltage decreases past the cutoff frequency is also based upon the reactance of the capacitor. In essence, the resistor and capacitor in a filter form a voltage divider in which the impedance of one of the components, the capacitor changes with frequency. Well past the cutoff frequency, the phase of the complex resultant of the math involved swings almost entirely to the capacitor, so, in rough terms, the output amplitude is dependent on the reactance of the capacitor. For example, an increase in frequency of 10 (a decade) results in a decrease in capacitive reactance to 1/10th, and the output of the filter will change by either an increase of 10x (HPF) or a decrease to 1/10th (LPF). Converting 10 to dB is 20 dB, and 1/10th is -20 dB. Therefore, a circuit with only one capacitor will have a **Rolloff** in the reject band with a magnitude of 20 dB/decade -- rising for a HPF and falling for a LPF.

Question:

A particular low pass filter has a "corner frequency" (cutoff frequency) of 3 kHz. If the amplitude of its output signal is 5 $V_p$ at 1 kHz (that's in the passband), what is the expected amplitude at the cutoff frequency, 3 kHz? 3.54 $V_p$

Since this filter has only one capacitor, at what frequency would we expect the signal amplitude to drop by 20 dB from the passband amplitude? 30 kHz.

What would the signal amplitude be at a frequency one decade above the cutoff frequency? 500 $mV_p$

If the resistor in this filter is 10 k$\Omega$, what must the capacitance be? 5.31 nF.